1. Mathematical Aspects

The topic of this year's workshop is symplectic fillings of contact manifolds, an area at the intersection of symplectic and contact topology. More precisely, we will study the classification of symplectic manifolds with fixed contact boundary and use this to investigate exotic contact structures.

Symplectic geometry is the study of symplectic manifolds, which are smooth manifolds endowed with a closed, nondegenerate two form. Originally arising from Hamiltonian mechanics, symplectic manifolds are natural generalizations of cotangent bundles of smooth manifolds. There are many examples of symplectic manifolds arising from affine and projective algebraic geometry. Contact manifolds are in some sense the odd-dimensional version of symplectic manifolds. Examples include 1-jet spaces and odd-dimensional spheres.

Symplectic and contact topology have both *flexible* and *rigid* phenomena. The flexible side is governed by h-principles, which reduce symplectic topology to algebraic and differential topology. One of the first h-principles was proven by Gromov [11] for isotropic submanifolds that are *subcritical*, i.e. have dimension *less* than half the dimension of the ambient symplectic or contact manifold. This led to an h-principle for *subcritical* Weinstein domains [4], a certain class of exact symplectic manifolds that admit a symplectic handlebody decomposition with only subcritical index handles. Murphy [15] discovered that *loose* Legendrians also satisfy an h-principle, the first example of flexibility in the critical setting. Using Murphy's h-principle, Cieliebak and Eliashberg [4] extended the h-principle for subcritical Weinstein domains to *flexible* Weinstein domains, symplectic handlebodies whose critical half-dimensional handles are attached along loose Legendrians. In the contact setting, Eliashberg proved an h-principle for 3-manifolds: any 3-manifold with an almost complex structure on its stabilized tangent bundle admits a contact structure unique up to contactomorphism. Such contact structures are called *overtwisted*; contact structures that are not overtwisted are called *tight*. Recently, Borman-Eliashberg-Murphy [3] extended this h-principle to higher dimensions.

The rigid side emphasizes the difference between symplectic and algebraic topology. The main tools here are the ground-breaking theory of pseudo-holomorphic curves discovered by Gromov [10] and related invariants like symplectic homology [8, 20]. These invariants can often distinguish exotic symplectic and contact structures that cannot be distinguished via algebraic topology.

In certain cases, exact symplectic manifolds have a natural contact structure on their boundary, which case we say that the symplectic manifold is a filling of the contact manifold. Any contact manifold with an exact symplectic filling is tight [6, 10]. Currently, a major problem is symplectic and contact topology is to classify symplectic fillings of a given contact manifold. One of the main tools used to study this problem are open book decompositions of contact manifolds, i.e. a decomposition of the contact manifold into a mapping torus of some Weinstein domain via an exact symplectomorphism and smaller pieces. Giroux [9] showed that any contact manifold admits an open book decomposition and that in dimension three, this decomposition is essentially unique. Lefshetz fibrations induce natural open book decompositions on their contact boundaries and give rise to a factorization of the symplectomorphism monodromy of the OBD into right-handed Dehn twists. Conversely, any factorization of the symplectomorphism monodromy of a contact manifold OBD into right-handed Dehn twists yields a Lefschetz filling of the contact manifold. This idea led to the construction of contact 3-manifolds with infinitely many different symplectic fillings, e.g. fillings that are not homeomorphic [18] or are homeomorphic but not diffeomorphic [1]. This was recently extended to higher dimensions by Oba [17]. In dimension three, Wendl [22] used J-holomorphic curve techniques to reduce the classification of symplectic fillings of contact manifolds with *planar* open book decompositions to the more concrete problem of factorizing elements of the mapping group of surfaces into positive Dehn twists. Using this result, Plamenevskaya and Van Horn-Morris [19] gave a full classification of symplectic fillings of certain contact 3-manifolds.

Another general phenomenon that highlights the interplay of flexible and rigid sides is that contact manifolds with fillings from the flexible side of symplectic topology are themselves rigid and remember the topology of their fillings. For example, Eliashberg, Floer, and McDuff [14] used the technique of "filling by J-holomorphic curves" to prove that all exact symplectic fillings of $(S^{2n-1}, \xi_{std}) = \partial(B^{2n}, \omega_{std})$ are diffeomorphic to B^{2n}. This was extended in recent work of Barth-Geiges-Stipsicz [2] and Oancea-Viterbo [16] to contact boundaries of subcritical Weinstein domains: if (Y, ξ) is simply-connected and has a subcritical Weinstein filling W, then *all* exact symplectic fillings of (Y, ξ) are diffeomorphic to W. Because of the relation between symplectic fillings and factorizations of symplectomorphisms into right-handed Dehn twists, this result implies that any composition of right-handed Dehn twists on a Liouville domain of dimension at least four is never isotopic to the identity within the group of compactly supported symplectomorphisms [2]. Beyond the subcritical case, Lazarev [12] used Murphy's h-principle [15] and positive symplectic homology to prove that all flexible fillings of a fixed contact manifold have the same cohomology. In the non-flexible case, Li-Mak-Yasui [13] used the theory of symplectic caps to show that all exact fillings of unit cotangent bundles of surfaces have the same homology.

A related problem is to classify all contact structures on a fixed manifold with the same *almost contact class*, or differential-topology data. In fact, the most general way to construct tight contact structures in high dimensions is to realize them as the boundary of some exact symplectic manifold (although there are examples of contact manifolds that tight but not fillable). Furthermore, the set of all symplectic fillings of a given contact manifold is a contact invariant. Using this idea, Eliashberg [7] constructed the first exotic contact structures in high-dimensions. More precisely, he constructed contact structures on S^{4k+1} that have symplectic fillings not diffeomorphic to B^{4k+2}, which are therefore exotic by he Eliashberg-Floer-McDuff theorem. Later, Ustilovsky [21] showed that S^{4k+1} has infinitely many exotic contact structures in the standard almost contact class. Using the classification of flexible fillings, Lazarev [12] constructed many new examples of exotic contact structures: in dimension at least 5, any almost contact manifold with an almost Weinstein filling has infinitely many different contact structures. This gives a purely topological condition for contact manifolds to have infinitely many different contact structures. In dimension three, the situation is drastically different and many almost contact classes have only finitely many different contact structures [5].

Two of the core problems of this workshop are: (1) understanding fillings of contact manifolds in terms of Weinstein presentation of a given filling and (2) constructing examples of exotic contact structures. Both problems have seen important recent progress that we would like to explore in this workshop. Some of the results we plan to highlight are:

- Oba [17] used factorization of symplectic mapping class group elements to construct high-dimensional contact manifolds with infinitely many different symplectic fillings.
- Lazarev [12] showed all flexible fillings of a contact manifold have the same cohomology and constructed infinitely many contact structures for a large class of manifolds.
- Li, Mak, and Yasui [13] used symplectic caps to show that all exact fillings of unit cotangent bundles of surfaces have the same homology, an example of rigidity for non-flexible contact manifolds.
- Barth, Geiges, and Stipsicz[2] used filling by J-holomorphic curves to show that all exact symplectic fillings of contact manifolds that have a subcritical Weinstein filling are diffeomorphic and obtain restrictions of factorizations in the symplectic mapping class group.

Although this workshop focuses much more on contact manifolds, the topics in this workshop have some overlap with topics in last year's workshop - flexibility, Lefschetz fibrations, Dehn twists, and symplectic homology - allowing us to further develop the ideas discussed last year.

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